# Sampling-based Volumetric Methods for Optimal Feedback Planning

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### Abstract

We present a sampling-based, asymptotically optimal feedback planning method for the shortest path problem among obstacles in $\mathbb{R}^d$. Our method combines an incremental sampling-based Delaunay triangulation with the newly introduced Repairing Fast Marching Method for computing a converging sequence of control policies. The convergence rate and asymptotic computational complexity of the algorithm are proven theoretically. In addition, the proposed method is compared with the state-of-the-art asymptotically optimal path planners in numerical simulation of a realistic planning problem. Finally, we present a straightforward extension of our method that handles dynamic environments where obstacles can appear, disappear, or move.

## Introduction

The optimal navigation in environments with obstacles enables robots to perform tasks efficiently and in timely manner. Efficiency of many autonomous systems is critical for their successful deployment. For example, the profit margin of an autonomous transportation system is improved if the total distance traveled by each constituting vehicle is reduced. Increasing autonomy of such systems raises the demand for optimal robot motion planning algorithms.

The majority of previous optimal planning algorithms compute the lowest cost geometric path between robot's initial position and the goal. Typically, these *path-centric* algorithms use a Rapidly-exploring Random Graph (RRG) structure that approximates a set of all feasible path between the initial robot position and the goal in incremental fashion (Karaman and Frazzoli 2011). The optimal path is then computed using graph search algorithms on RRG. However, navigating along geometric paths is a difficult task for a robot due to random disturbances, low-fidelity dynamical models, and control signal saturation. Thus, separate path-following controllers are usually implemented, which inevitably introduce suboptimal motions and increase the complexity of robotic systems in general.

In this presentation, we propose computing a feedback control policy, which can be applied directly for robot navigation without restricting its motions onto one-dimensional graphs. The proposed *policy-centric* algorithm is build on fundamentally different discretization principles: 1) constructing an incremental volumetric cell decomposition (which we call a *mesh*) on the free space, $X_{\mathrm{free}}$, and 2) computing an optimal cost-to-go function approximation on this mesh using the novel Repairing Fast Marching Method (ReFMM), a modification of a popular numerical Hamilton-Jacobi-Bellman (HJB) solver (Sethian 1999) that avoids infinite cost-propagation loops on unstructured meshes. The proposed incremental mesh parallels Probabilistic RoadMaps (PRM) (Kavraki et al. 1996) in that it captures the topology of $X_{\mathrm{free}}$ and has the same asymptotic computational complexity as its optimal implementation, PRM$^*$ (Karaman and Frazzoli 2011). Unlike the PRM and PRM$^*$, this construction enables computing a stabilizing controller that navigates a robot through the volume of discretization cells instead of constraining its motions to edges of an 1D graph; see Figure 1.

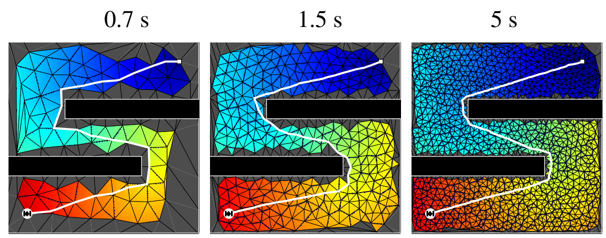


Figure 1: The cost-to-go function (color) and the resulting path (white) computed by ACIDIC algorithm for a robot (lower left) that desires to reach the goal (upper right) while avoiding obstacles (black).

Previously, numerical HJB solvers were deemed infeasible for large scale optimal planning problems mostly due to using fixed Cartesian meshes. Contrary to this popular belief, we establish that the proposed algorithm has the same asymptotic behavior as RRT$^*$, that is $O(\log(N))$ time complexity per iteration and $O((\log(N)/N)^{1/d})$ convergence rate with respect to the node number, $N$, and the dimension number, $d$. Moreover, numerical simulations show almost identical convergence towards the optimal path of our algorithm and one of the fastest graph-based planning algorithm, RRT$^{\#}$ (Arslan and Tsiotras 2013). With complexity and convergence results being equal, the benefit of using our algorithm is in the computed policy that can be used to control robot motions directly and avoid using path-following middleware.

## Asymptotically Optimal Feedback Planning

We now present the Asymptotically-optimal Control over Incremental Delaunay sImplicial Complexes (ACIDIC) algorithm for optimal feedback planning. The execution trace of our algorithm is similar to most sampling-based path-centric planners:

1. Sample vertex $x_{\mathrm{new}}$ from the configuration space, $X$;
2. Refine the Delaunay triangulation to include $x_{\mathrm{new}}$;
3. Update the cost-to-go values and the associated control policy in the simplicial approximation of $X_{\mathrm{free}}$.

At a conceptual level, the ACIDIC method "etches" $X_{\mathrm{free}}$ from $X$ while simultaneously refining a feedback control using a numerical HJB solver. In the limit of infinitely many sampled vertices, all points of $X_{\mathrm{free}}$ become part of the triangulation and the optimal feedback control is computed. We now discuss every step of the algorithm in details.

### Sampling Methods

Virtually any sampling strategy that has been developed for path-centric planners is suitable for our policy-centric method as well. These strategies include convergence accelerating techniques such as branch and bound methods, goal biasing, and so on. In this work, we assume samples are taken uniform at random and leave the discussion of advanced sampling techniques for future.

### Incremental Delaunay Triangulation

For every newly inserted point, the incremental Delaunay triangulation algorithm replaces cells that violate empty circumsphere property with new Delaunay cells. This update is analogous to local rewiring in the RRG algorithm. Using theoretical results on Poisson Delaunay triangulations in $\mathbb{R}^d$ (Miles 1974), we establish that, on average, $O(1)$ simplices are updated at each step.

After the DT is updated, a black box collision-detection module is used to find free-to-traverse simplices in the current triangulation. We implement the conservative collision-detection based on empty circumsphere, that is, the simplex is considered collision-free if its circumsphere is in $X_{\mathrm{free}}$.

### Repairing Fast Marching Method (ReFMM)

Our modifications to the FMM are aimed at preventing infinite update loops and costly updates in the entire $X_{\mathrm{free}}$. Borrowing ideas from replanning path-centric strategies, such as RRT$^{\mathrm{X}}$ (Otte and Frazzoli 2014), the ReFMM algorithm interrupts wavefront propagation when relative cost-to-go changes are smaller than a given parameter $\varepsilon$. We establish that ReFMM sacrifices the accuracy up to $\varepsilon$ relative error and takes, on average, $O(\log(N))$ amortized time per step to compute.

A peculiar side effect of our ReFMM implementation is a straightforward extension to an optimal feedback replanning algorithm, which allows fast feedback repairs when obstacles appear, disappear, or move.

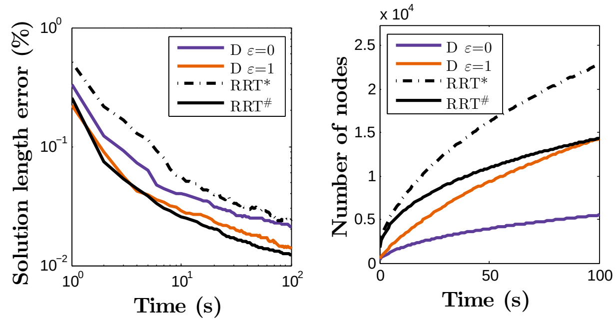


Figure 2: Averaged over 50 trials relative path error (left) and the node number (right) against running time for ACIDIC ("D") with $\varepsilon = 0\%$ and $\varepsilon = 1\%$, RRT$^*$ (black dot-dash), and RRT$^\#$ (black solid) in a 2D environment with obstacles.

## Numerical Simulations and Results

In the numerical simulations, we investigated the convergence of ACIDIC and compared it with that of RRT$^*$ and RRT$^\#$. In Figure 2, we plot the convergence and the number of nodes used by each algorithm with respect to running time. We conclude that ACIDIC with $\varepsilon = 1\%$ and RRT$^\#$ converge at the same rate and both significantly outperform RRT$^*$ algorithm (note the log-log plot). Moreover, ACIDIC uses fewer vertices to compute the same quality solution as RRT$^\#$, which is beneficial for high-cost collision detection modules.

It is difficult to illustrate using figures replanning capabilities of our algorithm. Thus, we present short movies at `http://tinyurl.com/qjnazvr`.

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